

Section 1-8 Radical Rules

PRODUCT RULE:

$$\sqrt[a]{x} \cdot \sqrt[a]{y} = \sqrt[a]{xy}$$

Example:

$$\sqrt{10} \cdot \sqrt{x} = \sqrt{10x}$$

QUOTIENT RULE:

$$\frac{\sqrt[a]{x}}{\sqrt[a]{y}} = \sqrt[a]{\frac{x}{y}}$$

Example:

$$\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$$

More directly, when determining a product or quotient of radicals and the indices (the small number in front of the radical) are the same then you can rewrite 2 radicals as 1 or 1 radical as 2.

Simplify by rewriting the following using only one radical sign (i.e. rewriting 2 radicals as 1).

$$\begin{aligned} 1. \quad \sqrt{3} \cdot \sqrt{12} &= \sqrt{3 \cdot 12} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

6

$$2. \quad \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

2

$$\begin{aligned} 3. \quad \sqrt{7x} \cdot \sqrt{2y} \\ \sqrt{7x \cdot 2y} &= \sqrt{14xy} \end{aligned}$$

$\sqrt{14xy}$

$$\begin{aligned} 4. \quad \frac{\sqrt[3]{12x^2}}{\sqrt[3]{4x}} &= \sqrt[3]{\frac{12x^2}{4x}} = \sqrt[3]{\frac{12 \cdot \cancel{x} \cdot x}{4 \cdot \cancel{x}}} \\ &= \sqrt[3]{3x} \end{aligned}$$

$\sqrt[3]{3x}$

Simplify by rewriting the following using multiple radical sign (i.e. rewriting 1 radical as 2).

$$5. \quad \sqrt{\frac{144}{25}} = \frac{\sqrt{144}}{\sqrt{25}} = \frac{12}{5}$$

$\frac{12}{5}$

$$6. \quad \sqrt{\frac{x^6}{121}} = \frac{\sqrt{x^6}}{\sqrt{121}} = \frac{x^3}{11}$$

$$x^3 \cdot x^3 = x^6$$

$\frac{x^3}{11}$

Express each radical in simplified form.

7. $\sqrt{48}$

$$= \sqrt{4 \cdot 4 \cdot 3}$$

$$= \sqrt{4} \cdot \sqrt{4} \cdot \sqrt{3}$$

$$= 2 \cdot 2 \cdot \sqrt{3}$$

Exact
 $4\sqrt{3}$ P

8. $\sqrt{450x^4y^5}$

$$= \sqrt{9 \cdot 25 \cdot 2 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y^2 \cdot y}$$

$$= \sqrt{9} \cdot \sqrt{25} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{y^2} \cdot \sqrt{y^2} \cdot \sqrt{2y}$$

$$= 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot \sqrt{2y}$$

$$= 15x^2y^2\sqrt{2y}$$

$15x^2y^2\sqrt{2y}$ N

9. $\sqrt{72x^5y^6}$

$$= \sqrt{9 \cdot 4 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y^2 \cdot y^2 \cdot 2x}$$

$$= \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{y^2} \cdot \sqrt{y^2} \cdot \sqrt{y^2} \cdot \sqrt{2x}$$

$$= 3 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot \sqrt{2x}$$

$$= 6x^2y^3\sqrt{2x}$$

$6x^2y^3\sqrt{2x}$ O

10. $\sqrt{300x^{12}}$

$$= 2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot \sqrt{3}$$

$10x^6\sqrt{3}$ O

11. $\sqrt{675x^4y^{11}}$

$$= 5 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot \sqrt{3y}$$

$15x^2y^5\sqrt{3y}$ N

12. $-\sqrt{81x^3y^8}$

$$= -3 \cdot 3 \cdot x \cdot y \cdot y \cdot y \cdot y \cdot \sqrt{x}$$

$-9x^4\sqrt{x}$ A

13. $\sqrt[3]{48x^7y^3}$

$$= \sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^3} \cdot \sqrt[3]{6x}$$

$$= 2 \cdot x \cdot x \cdot y \cdot \sqrt[3]{6x}$$

$2x^2y\sqrt[3]{6x}$ N

14. $\sqrt[3]{81x^{10}y^3}$

$$= 3 \cdot x \cdot x \cdot x \cdot y \cdot \sqrt[3]{3x}$$

$3x^3y\sqrt[3]{3x}$ I

15. $\sqrt[3]{-27x^5}$

$$= -3x \cdot \sqrt[3]{x^2}$$

$-3x\sqrt[3]{x^2}$ K

Use the letters and answers to match the answer to the riddle. Only some answers will be used.

"What is an opinion without π ?"

A

N

O

N

I

O

N

$-9xy^4\sqrt{x}$

$15x^2y^2\sqrt{2y}$

$10x^6\sqrt{3}$

$2x^2y\sqrt[3]{6x}$

$3x^3y\sqrt[3]{3x}$

$6x^2y^3\sqrt{2x}$

$15x^2y^5\sqrt[3]{3y}$

Express each radical in simplified form.

$$16. \sqrt{6x} \cdot \sqrt{12x} = \sqrt{6x \cdot 12x} = \sqrt{72x^2}$$

$$\begin{array}{c} \sqrt{72x^2} \\ \swarrow \searrow \\ 7 \cdot 8 \\ \swarrow \searrow \swarrow \searrow \\ 3 \cdot 3 \cdot 4 \cdot 2 \\ \swarrow \searrow \swarrow \searrow \swarrow \searrow \\ 3 \cdot 3 \cdot 2 \cdot 2 \cdot x \cdot x \\ \underline{\underline{3 \cdot 2 \cdot x \sqrt{2}}} \end{array}$$

$$6x\sqrt{2}$$

$$17. \sqrt{18a^5} \cdot \sqrt{6a^4} = \sqrt{18a^5 \cdot 6a^4} = \sqrt{108a^9}$$

$$\begin{array}{c} \sqrt{108a^9} \\ \swarrow \searrow \\ 6 \cdot 18 \\ \swarrow \searrow \swarrow \searrow \\ 2 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a \\ \swarrow \searrow \swarrow \searrow \swarrow \searrow \swarrow \searrow \\ 2 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot a \sqrt{3a} \end{array}$$

$$6a^4\sqrt{3a}$$

Simplify. Assume that all variable represent positive real numbers.

$$18. 5\sqrt{3} + \sqrt{12} - 2\sqrt{3} + 4\sqrt{2}$$

$$\begin{array}{c} \sqrt{12} \\ \swarrow \searrow \\ 2 \cdot 3 \\ \swarrow \searrow \\ 2 \cdot \sqrt{3} \end{array}$$

$$3\sqrt{3} + 5\sqrt{2}$$

$$3\sqrt{3} + 5\sqrt{2}$$

$$19. \sqrt{108} + 5\sqrt{12} - 4\sqrt{44}$$

$$\begin{array}{c} \sqrt{108} \quad \sqrt{12} \quad \sqrt{44} \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot 3 \cdot 3 \cdot 2 \quad 2 \cdot 3 \quad 2 \cdot 11 \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot 3 \sqrt{3} \quad 2 \cdot 3 \quad 2 \cdot \sqrt{11} \\ \underline{\underline{6\sqrt{3} + 10\sqrt{3} - 8\sqrt{11}}} \end{array}$$

$$16\sqrt{3} - 8\sqrt{11}$$

$$20. 2\sqrt{150} + \sqrt{18} + 3\sqrt{8} - \sqrt{24}$$

$$\begin{array}{c} \sqrt{150} \quad \sqrt{18} \quad \sqrt{8} \quad \sqrt{24} \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot 3 \cdot 5 \cdot 2 \quad 2 \cdot 3 \cdot 3 \quad 2 \cdot 2 \cdot 2 \quad 2 \cdot 2 \cdot 3 \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot 5 \sqrt{6} \quad 3 \sqrt{2} \quad 2 \sqrt{2} \quad 2 \sqrt{6} \\ \underline{\underline{10\sqrt{6} + 3\sqrt{2} + 6\sqrt{2} - 2\sqrt{6}}} \end{array}$$

$$8\sqrt{6} + 9\sqrt{2}$$

$$21. y\sqrt{18} - 3\sqrt{12y^4} + 2\sqrt{8y^2}$$

$$\begin{array}{c} \sqrt{18} \quad \sqrt{12y^4} \quad \sqrt{8y^2} \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 3 \cdot 2 \cdot 3 \quad 2 \cdot 3 \cdot 2 \cdot y \cdot y \quad 2 \cdot 2 \cdot 2 \cdot y \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ y \cdot 3 \cdot \sqrt{2} \quad 3 \cdot 2 \cdot y \cdot y \sqrt{3} \quad 2 \cdot 2 \cdot y \sqrt{2} \\ \underline{\underline{3y\sqrt{2} - 6y^2\sqrt{3} + 4y\sqrt{2}}} \end{array}$$

$$7y\sqrt{2} - 6y^2\sqrt{3}$$

$$22. x\sqrt{32x^2} + 2\sqrt{18x^4}$$

$$\begin{array}{c} \sqrt{32x^2} \quad \sqrt{18x^4} \\ \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \quad 3 \cdot 3 \cdot 2 \cdot x \cdot x \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ x \cdot 2 \cdot 2 \cdot x \sqrt{2} \quad 2 \cdot 3 \cdot x \cdot x \sqrt{2} \\ \underline{\underline{4x^2\sqrt{2} + 6x^2\sqrt{2}}} \end{array}$$

$$10x^2\sqrt{2}$$

$$23. 5\sqrt{18x^4} - 3x\sqrt{8x^2} - x^2\sqrt{2}$$

$$\begin{array}{c} \sqrt{18x^4} \quad \sqrt{8x^2} \quad \sqrt{2} \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 3 \cdot 3 \cdot 2 \cdot x \cdot x \quad 2 \cdot 2 \cdot 2 \cdot x \quad x \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 5 \cdot 3 \cdot x \cdot x \sqrt{2} \quad 3 \cdot x \cdot 2 \cdot x \sqrt{2} \quad -x^2\sqrt{2} \\ \underline{\underline{15x^2\sqrt{2} - 6x^2\sqrt{2} - x^2\sqrt{2}}} \end{array}$$

$$8x^2\sqrt{2}$$

$$24. 2\sqrt{10}(\sqrt{36} + 2\sqrt{5} - \sqrt{24})$$

$$\begin{array}{c} \sqrt{36} \quad \sqrt{5} \quad \sqrt{24} \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 6 \cdot 6 \quad 5 \cdot 5 \quad 2 \cdot 2 \cdot 3 \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 6 \cdot 2 \sqrt{15} \quad 4 \cdot 5 \sqrt{2} \quad 2 \cdot 2 \cdot 2 \sqrt{15} \\ \underline{\underline{12\sqrt{15} + 20\sqrt{2} - 8\sqrt{15}}} \end{array}$$

$$4\sqrt{15} + 20\sqrt{2}$$

$$25. \sqrt{2x}(\sqrt{6x} + 3\sqrt{x})$$

$$\begin{array}{c} \sqrt{2x} \quad \sqrt{6x} \quad \sqrt{x} \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot x \quad 2 \cdot 3 \cdot x \quad x \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot x \sqrt{3} \quad 3 \cdot x \sqrt{2} \\ \underline{\underline{2x\sqrt{3} + 3x\sqrt{2}}} \end{array}$$

$$2x\sqrt{3} + 3x\sqrt{2}$$

$$26. 3\sqrt{6a}(\sqrt{4a} + 2\sqrt{15a^2})$$

$$\begin{array}{c} \sqrt{4a} \quad \sqrt{15a^2} \\ \swarrow \searrow \quad \swarrow \searrow \\ 2 \cdot 2 \cdot a \quad 3 \cdot 5 \cdot a \cdot a \\ \swarrow \searrow \quad \swarrow \searrow \quad \swarrow \searrow \\ 3 \cdot 2 \cdot a \sqrt{6} \quad 6 \cdot 3 \cdot a \sqrt{10a} \\ \underline{\underline{6a\sqrt{6} + 18a\sqrt{10a}}} \end{array}$$

$$6a\sqrt{6} + 18a\sqrt{10a}$$

Simplify. Assume that all variable represent positive real numbers and rationalize all denominators.

$$18. \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$$

3√(5)
1.341640786
3√(5)/5
1.341640786
■

$$\frac{3\sqrt{5}}{5}$$

$$21. \sqrt{\frac{16}{27}} = \frac{\sqrt{16}}{\sqrt{27}} = \frac{4}{3\sqrt{3}}$$

$$= \frac{4}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3\sqrt{9}}$$

$$= \frac{4\sqrt{3}}{3 \cdot 3} = \boxed{\frac{4\sqrt{3}}{9}}$$

$$\frac{4\sqrt{3}}{9}$$

$$19. \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$2\sqrt{3}$$

$$22. \frac{(\sqrt{12} + 8\sqrt{3} - 2\sqrt{27}) \cdot \frac{\sqrt{2}}{\sqrt{2}}}{\sqrt{2}}$$

$$\frac{\sqrt{24} + 8\sqrt{6} - 2\sqrt{54}}{2}$$

$$\frac{2\sqrt{6} + 8\sqrt{6} - 6\sqrt{6}}{2}$$

$$\frac{4\sqrt{6}}{2} = 2\sqrt{6}$$

$$2\sqrt{6}$$

$$20. \frac{3\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{12}}{\sqrt{36}} = \frac{3\sqrt{12}}{6} = \frac{\sqrt{12}}{2}$$

$$= \frac{\sqrt{4 \cdot 3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

3√(2)/√(6)
1.732050808
√(3)
1.732050808

$$\sqrt{3}$$

$$23. \frac{\sqrt{2}(\sqrt{12} - \sqrt{3})}{\sqrt{3}}$$

$$= \frac{(\sqrt{24} - \sqrt{6}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

$$= \frac{\sqrt{72} - \sqrt{18}}{\sqrt{9}} = \frac{\sqrt{72} - \sqrt{18}}{3}$$

$$= \frac{6\sqrt{2} - 3\sqrt{2}}{3}$$

$$= \frac{3\sqrt{2}}{3} = \sqrt{2}$$

$$\sqrt{72} \rightarrow 9 \cdot 8 \rightarrow 3 \cdot 3 \cdot 4 \cdot 2 \rightarrow 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$$

$$3 \cdot 2 \cdot 2$$

$$\sqrt{18} \rightarrow 9 \cdot 2 \rightarrow 3 \cdot 3 \cdot 2$$

$$\sqrt{2}$$